SOLIDS SEGREGATION IN SLURRY FLOW THROUGH A T-JUNCTION WITH A HORIZONTAL APPROACH

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Abstract—Particle segregation in slurry flow through a T-junction with a horizontal approach has been examined. Water-sand slurries were used with T-junctions of various orientations: upward and downward in a vertical plane and side orientation in a horizontal plane. The effects of particle size, upstream solids concentration, upstream bulk velocity and flow ratio (defined as branch flow rate to upstream flow rate) on the branch and run solids concentration were studied.

For given upstream conditions, the experimental results showed that for all branch orientations the concentration ratios of the branch and run (defined as the ratio of the branch or run solids concentration to the upstream solids concentration) are strong functions of the flow ratio. For the vertical upward and side orientations, the concentration ratio was found to be less than unity and approaches one as the flow ratio approaches unity. For the vertical downward orientation, it was found that the concentration ratio was mainly greater than unity and approached unity as the flow ratio approached one.

For all orientations, the branch concentration ratio was found to be a function of the upstream bulk velocity, solids concentration and particle size. For the vertical upward orientation, the concentration ratio increased as the upstream velocity or solids concentration increased, but decreased as the particle size increased. The opposite trends occurred for the vertical downward orientation. For the side orientation, the branch concentration ratio decreased as the upstream velocity, particle size increased, whereas the effect of upstream solids concentration on the branch concentration ratio was found to be insignificant.

Key Words: solids segregation, slurry manifold, T-junction, slurry flow

INTRODUCTION

Flow of slurries through T-junctions and manifolds is commonly encountered in many industrial applications, e.g. distribution of coal-oil slurry into parallel preheaters in the EDS coal liquifaction process (Segev & Kern 1985). A potential problem in such configurations is maldistribution of coal particles between the preheater passes. This could lead to operation inefficiency due to unequal heat transfer to each pass. Other applications include sand distribution in perforated casings used in oilfields (Haynes & Gray 1974; Gruesbeck & Collins 1982), passage of fibres through slots in pressure screens (Gooding 1986) and separation of red cells in the side branches of bifurcations in the smaller vessels in the circulatory system (Bugliarello & Hsiao 1964).

Due to the difference in inertia between the liquid and solid phases or uneven solids distribution upstream of a bifurcation point, the branch has different solids concentration from that in the main pipe. There are two criteria for assessing the degree of separation that occurs in a branch. The first criterion is the concentration ratio or the separation ratio, S_i , defined as

$$S_i = \frac{C_i}{C_0}$$
 (with $i = 1$ for the branch and $i = 2$ for the run), [1]

where C_1 is the branch solids concentration, C_2 is the solids concentration downstream of the T-junction, i.e. in the run, and C_0 is the solids concentration in the main pipe upstream of the branch. This criterion can be used when designing a T-junction or a manifold based on equal solids concentration in the branches. The second criterion is the transport efficiency, E_i , defined as the mass flow rate of the particles in the branch or in the run to the mass flow rate of solids in the main pipe. E_i can be expressed as

$$E_i = \frac{C_i Q_i}{C_0 Q_0},$$
 [2]

where C_i is the solids concentration in the branch or in the run, C_0 is the upstream solids concentration in the main pipe, Q_i is the branch or the run flow rate and Q_0 is the main pipe flow rate. This measure can be used when designing a T-junction or a manifold based on equal solids flow rate in the branches.

Although slurry flow through T-junctions has many practical applications, little is known about solids distribution in the branches, especially those of a horizontal approach. Iwanami & Suu (1969a, b) were the first to measure pressure losses for slurry flow through right-angled branches, but no concentration measurements were undertaken. Bugliarello & Hsiao (1964) studied phase separation of neutrally buoyant particles using branches of different sizes at various angles to the main flow. The flow in the main pipe (vertical) was laminar in a downward direction. Although the effect of inertia can be neglected in their system, the branch concentration ratio was found to be generally less than unity and increased as the branch flow ratio increased. This rather unexpected result was explained in terms of uneven concentration and velocity profiles upstream of the bifurcation point.

Haynes & Gray (1974) experimentally studied particle transport in a perforated casing of 4" dia. The slurry (water-sand) flow upstream of the perforations was vertically downward. They found the branch concentration ratio to be less than unity. The branch concentration was low for the coarser particles, especially at higher upstream solids concentrations. Also, the effect of the branch size on the concentration ratio was found to be insignificant. The amount of the fine particles was larger in the branch than that in the main pipe.

Torrest & Savage (1975) studied the collection of particles in small branches. The flow in their experiments was vertically downward with a uniform concentration profile upstream of the branch. They found the branch transport efficiency to be independent of the branch size. The transport efficiency was larger for particles with small settling velocities and it increased at higher branch flow rates. The following empirical correlation was developed to estimate the branch transport efficiency in terms of particle settling velocity (V_t) and the upstream bulk velocity (U_0):

$$E_{1} = \frac{Q_{1}^{*}}{100} \left[\frac{40(V_{1} + U_{0}) - 58.4}{1 - 125(V_{1} + U_{0})} \right],$$
[3]

with Q_1^* (branch flow rate) in gpm and $(V_1 + U_0)$ in m/s. This correlation is valid for the range $0.4 > V_1 + U_0 > 0.04$.

Gruesbeck & Collins (1982) studied sand transport through a perforated casing. According to their experimental results, the branch transport efficiency was a linear function of the branch flow ratio (Q_1/Q_0) . For a given branch flow ratio, the branch transport efficiency increased as the particle size decreased. The effects of branch size and solids concentration on the branch transport efficiency were found to be insignificant.

Moujaes (1984) found that the branch concentration ratios were consistently lower than unity for slurries of 60/80 mesh sand. Higher branch concentration ratios were obtained with 140 mesh particles. These results were explained in terms of a non-uniform radial concentration profile in the main pipe (vertical) with a lower solids concentration near the wall of the pipe.

Nasr-El-Din *et al.* (1985) studied particle transport into small branches of a vertical approach. They found that the branch concentration ratio was always less than unity and increased as the branch flow ratio increased. They also noted that the amount of the fine particles in the branch was larger than that in the main pipe, especially at low branch flow ratios.

Segev & Kern (1985) were the first to study solid-liquid separation in a slurry manifold with a horizontal approach. A variable diameter manifold was designed to ensure that the slurry velocity was the same in each section of the header. The branches were of 3/4'' dia and were tested for side and downward orientations. The effects of the upstream bulk velocity and solids concentration on the branch concentration were examined. The efficiency of the branch was expressed in terms of a separation parameter, B_1 , defined as

$$\boldsymbol{B}_1 = \frac{X_1}{X_0} \tag{4}$$

where X_1 is the solids mass fraction in the branch and X_0 is the solids mass fraction in the main pipe upstream of the branch. They developed the following semi-empirical correlation for the branch separation parameter:

$$B_1 = 0.875 \,\alpha \, \frac{(C^*)^{0.356}}{K^{0.05}},\tag{5}$$

where α is a characteristic constant for the T-corner, $C^* = (C_{center}/C_0)$ for branches of side orientation and (C_{bottom}/C_0) for branches of downward orientation. C_{center} and C_{bottom} were obtained from predicted vertical concentration profiles using a modified diffusion-type model originally developed by Karabelas (1977) for slurries of very low solids concentrations. One should mention that [5] was obtained for a fixed branch to main flow mass flow ratio of 0.25 and hence its use is limited.

Nasr-El-Din & Shook (1986) studied particle segregation in slurry flow in vertical T-junctions with an area ratio of unity. Solids maldistribution was observed for all the branches examined, especially for coarse particles and at low branch flow ratios.

As can be seen, most of the previous studies were conducted for branches of a vertical approach, where the solids concentration and velocity profiles are nearly uniform. Consequently, one can directly relate the branch concentration to the upstream conditions. However, for branches of a horizontal approach, estimating branch concentration from upstream conditions is much more complicated. For slurry flow in a horizontal pipe the concentration and velocity profiles are not uniform, especially for settling slurries. These profiles are functions of solids concentration, bulk velocity, particle settling velocity and pipe diameter. Predicting these profiles is not an easy task. However, starting from the linear momentum equations of the two phases, Roco & Shook (1981, 1984) were able to develop a model that predicts these profiles fairly well. Both experimental measurements and model predictions have shown that the concentration profile of settling slurries is asymmetric: solids concentration is high at the bottom of the pipe (in the absence of a positive concentration gradient) and monotonically decreases as one approaches the top of the pipe. Local particle velocity measurements (Brown & Shook 1982) have shown that particle velocity exhibits a maximum above the centre of the pipe. Other local concentration measurements (Nasr-El-Din et al. 1987) have shown that even for non-settling slurries the concentration profile is not uniform, except for very fine particles at low solids concentrations.

The objectives of the present investigation are to study solids distribution in a T-junction with a horizontal approach and to examine the effects of upstream conditions on the branch and run solids concentration ratios for three branch orientations: upward and downward in a vertical plane and side orientation in a horizontal plane.

PARTICLE EQUATION OF MOTION

It is instructive to discuss the effects of the various forces on the particle motion before examining the experimental results. The linear momentum equation for the solids is (Wallis 1969):

$$\rho_{\rm s}a_{\rm s} = -\nabla p + \rho_{\rm s}g + f_{\rm sf} + f_{\rm sw},\tag{6}$$

where a_s is the particle acceleration, p is the fluid pressure, f_{sf} is the drag force exerted by the fluid on solids, f_{sw} is the wall friction force acting on the solids and g is the gravitational acceleration. To simplify the analysis, we adopt the Lagrangian frame of reference and neglect the effect of solids on the fluid flow field. Consequently, this analysis will be valid for fluid-solid systems of low solids concentrations.

According to Wallis (1969), the drag force exerted by the fluid on the solids is

$$f_{\rm sf} = \frac{\left(\frac{3}{4}\right) C_{\rm DS}\left(\frac{\rho_{\rm f}}{\rho_{\rm s}}\right) |v_{\rm f} - v_{\rm s}| (v_{\rm f} - v_{\rm s})}{(1 - C)^n},\tag{7}$$

where v_f and v_s are the local fluid and particle velocities, ρ_f and ρ_s are the densities of the fluid and the solids, respectively, C is the local solids concentration, C_{DS} is the drag coefficient and n = 1.7 for non-flocculating spherical particles (Wallis 1969).

The drag coefficient is a function of the particle shape and the local particle Reynolds number (Re_s) . For spherical particles, the drag coefficient can be determined from the following relation (Wallis 1969):

$$C_{\rm DS} = \frac{24}{\text{Re}_{\rm s}} (1 + 0.15 \text{ Re}_{\rm s}^{0.687}), \quad \text{for } \text{Re}_{\rm s} \le 1000$$
[7a]
= 0.44, for $\text{Re}_{\rm s} \ge 1000$,

where Re_s is defined as

$$\operatorname{Re}_{s} = \frac{\rho_{f} d_{50} |v_{f} - v_{s}| (1 - C)}{\mu_{f}}.$$
[7b]

Neglecting the wall friction (f_{sw}) , [6] can be written in a dimensionless form as

$$\frac{\mathrm{d}v'_{\mathrm{s}}}{\mathrm{d}\tau} = -\frac{\rho_{\mathrm{f}}}{2\rho_{\mathrm{s}}}\nabla'p' + \frac{C_{\mathrm{DS}}\mathrm{Re}_{0}|v'_{\mathrm{f}} - v'_{\mathrm{s}}|(v'_{\mathrm{f}} - v'_{\mathrm{s}})}{24K(1-c)^{n}} + \frac{gD_{1}}{U_{0}^{2}},$$
[8]

where

$$v'_{s} = \frac{v_{s}}{U_{0}}, \quad v'_{f} = \frac{v_{f}}{U_{0}}, \quad \operatorname{Re}_{0} = \frac{\rho_{f} d_{50} U_{0}}{\mu_{f}}$$
$$\tau = \frac{t U_{0}}{D_{1}}, \quad \nabla' p' = \frac{2D_{1}}{\rho_{f} U_{0}^{2}} \nabla p$$

and K is the ratio of particle inertia to the fluid drag, known as the particle inertia parameter or the Stokes number, defined as

$$K = \frac{\rho_s \, d_{50}^2 \, U_0}{18\,\mu_{\rm f} D_1},\tag{9}$$

where ρ_s is the solids density, d_{50} is the mean particle diameter, U_0 is the upstream bulk velocity, μ_f is the fluid viscosity and D_1 is the branch diameter.

The branch concentration ratio can be determined from the particle trajectories which can be obtained by integrating [8]. For a T-junction with a vertical approach, where the concentration and velocity profiles are uniform, Nasr-El-Din & Shook (1986) determined the branch concentration ratio by integrating [8] using a two-dimensional potential flow. However, for a horizontal pipe the concentration profile is not uniform and to integrate [8] one has to know the flow field ahead of the branch. To date there is no adequate method to predict the flow field into side branches. The flow into side branches is strongly three-dimensional. This is because as the fluid changes direction, it generates a secondary flow. Iwanami & Suu (1969a, b) observed a helical motion in the branches, indicating the presence of secondary flow. Meckel (1974) measured velocity distributions downstream of T-junctions and elbows. His measurements confirmed the presence of two vortices downstream of the T-junctions.

Due to the complex nature of the flow into side branches, it is very difficult to determine branch concentration ratios. However, by examining [8], one can understand many of the trends observed in this study. The term on the l.h.s. of [8] represents the particle acceleration, the second term on the r.h.s. of [8] represents the fluid drag on the particle and the last term represents the effect of gravity. One observes from [8] that the effect of gravity on the particle acceleration decreases as the upstream bulk velocity increases. Considering the range of the bulk velocities tested in this study, the effect of the gravity term is small and can be neglected. Also, the drag force depends on the operating conditions. For a given flow ratio, [8] shows that when the particle inertia parameter is large, the drag force becomes negligible, leading to a negligible particle acceleration. Consequently, a solid particle cannot follow changes in the direction of the fluid flow. This leads to low branch solids concentration ratios, as a branch flow constitutes a change in the direction of the main pipe flow. When K approaches infinity, the particle trajectories approach straight lines leading to branch solids concentration of zero. On the other hand, when K is very small, the particle can experience a large acceleration which would enable it to change direction with the fluid flow into the branch. Consequently, as K approaches zero, the particle trajectories coincide with the fluid streamlines, and the branch concentration ratio approaches unity.

From an experimental point of view, the drag force can be increased by increasing solids concentration or decreasing the particle inertia parameter. The latter can be lowered by decreasing the particle size, upstream bulk velocity or by increasing the branch diameter.

One should mention that if the concentration profile upstream of the T-junction is uniform, [5] predicts that the branch concentration ratio equals zero as K approaches infinity. This agrees with the above analysis. However, as K approaches zero, [5] does not predict a value of unity for B_1 and consequently it should only be used within the experimental range for which it was derived.

EXPERIMENTAL STUDIES

Figure 1 shows a schematic diagram of the closed loop used in the experimental program. The loop consisted of 1" pipeline, a holding tank equipped with a stirrer, a Foxboro magnetic flowmeter (type 1801) of 1" i.d., a double pipe heat exchanger, and a 3 h.p. Moyno pump with a variable speed drive. The loop has a transparent section to allow for visual observation. A T-junction of equal diameters $(D_0 = D_1 = D_2 = 25.4 \text{ mm})$ was installed in the horizontal section of the loop. To minimize the effect of the secondary flow generated by an upstream elbow, straightening vanes of 10 cm length were inserted downstream of the elbow, as was suggested by Sharp & O'Neil (1971). In addition, the T-junction was installed 135 pipe diameters downstream of the elbow. T-junctions of various orientations were tested in this study: upward and downward in a vertical plane and side orientation in a horizontal plane. The flow rate of the slurry through the branch and the run was controlled by valves, as shown in figure 1. The loop has a sampling port for each branch and for the main pipe. Samples were collected over short timed intervals so that flow rates and concentrations can be calculated from the weights and volumes of the samples.

The dispersed phase used in this study was silica sand particles of density 2650 kg/m³ and the continuous phase was tap water. Three sand fractions of 0.08, 0.23, 0.4 mm mean diameter were used in this study, figure 2 shows their particle size distribution. Hereafter these sands will be referred to as fine, medium and coarse sand, respectively. The experiments covered the effects of the upstream solids concentration, particle size, upstream bulk velocity, flow ratio and branch orientation on the solids concentration in the branch (C_1) and the run (C_2) . Table 1 summarizes the ranges of the parameters covered in the present study.

During the course of an experimental run, the upstream bulk velocity, the upstream solids concentration and temperature $(22 \pm 1^{\circ}C)$ were kept constant.



Figure 1. Schematic diagram of the slurry loop.



Figure 2. Particle size distribution for sands.

Table 1. Parameters studied		
Parameter	Symbol	Range
Upstream solids concentration	C_0	6-20% by vol
Upstream bulk velocity	U_0	1.7-4.0 m/s
Branch angle	θ	90 °
Mean particle diameter	d_{s0}	0.08–0.4 mm
Particle inertia parameter	ĸ	0.06-3.7
Upstream particle Reynolds number	Reo	136-1600
Particle settling velocity	V _t	0.65-5.73 cm/s

RESULTS AND DISCUSSION

Upward orientation

Figures 3a and 3b show the effect of the flow ratio, Q_i/Q_0 on the concentration ratios, C_i/C_0 , i = 1, 2, with the particle size as a parameter. Slurries having a solids concentration of 12% and bulk velocity of 2.6 m/s were used with a T-junction with an upward orientation. One observes from figure 3a that the concentration ratio C_1/C_0 equals zero at a flow ratio (Q_1/Q_0) of zero, i.e. the branch valve is fully closed. The branch concentration ratio monotonically increases as the flow ratio increases and approaches unity as the flow ratio approaches one (the run valve is fully closed). These results are reasonable since the sand particles are heavier than water, i.e. they have a higher inertia per unit volume than water. This means that it is much easier for the fluid to change direction upon applying a pressure gradient normal to the main flow direction. As the normal pressure gradient increases, more fluid reports to the branch dragging more particles into the branch. Figure 3a also shows that the concentration ratio is a function of the particle size. For a given flow ratio, the coarser the particle, the lower the branch concentration ratio. The particle size also affects the rate of change of the concentration ratio with respect to the flow ratio, especially close to the end points of the flow ratio of zero and one. At a flow ratio of zero, the rate of change of the



Figure 3a. Effect of particle size on the branch concentration ratio for upward orientation at $C_0 = 12\%$ and $U_0 = 2.6$ m/s.



Figure 3b. Effect of particle size on the run concentration ratio for upward orientation at $C_0 = 12\%$ and $U_0 = 2.6$ m/s.

concentration ratio with respect to the flow ratio is high for the fine particles and decreases for the coarse particles. The opposite trend occurs at a flow ratio of unity where the rate of change of the concentration ratio is high for the coarse particles and decreases for the fine particles. The trends shown in figure 3a can be explained as follows. A coarse particle has a large particle inertia parameter, K, and consequently, its acceleration (according to [8]) is low. This means that the coarse particles cannot track changes in the fluid flow direction. In addition, the concentration profile upstream of the T-junction for the coarse particles is more asymmetric, with less solids in the upper half of the pipe. Obviously, both factors tend to lower the branch solids concentration.

Figure 3b shows the variation of the run solids concentration (i.e. downstream of the T-junction) as a function of the flow ratio (Q_2/Q_0) with the particle size as a parameter. In the limiting case where all the particles report to the run, i.e. the branch solids concentration equals zero, the run concentration ratio (i.e. the total separation curve) is given by

$$\frac{C_2}{C_0} = \left(\frac{Q_2}{Q_0}\right)^{-1}.$$
 [10]

The results shown in figure 3b indicate that the concentration ratio C_2/C_0 approaches the total separation limit as the particle size increases, especially at high run flow ratios. It is interesting to note that the run concentration ratio is mainly higher than unity. This indicates that a T-junction with an upward orientation has the effect of increasing the solids concentration downstream of the T-junction. One also observes a maximum in the run concentration ratio at approximately a run flow ratio of 0.1. The value of this maximum is a strong function of the particle size. The larger the particle size, the higher the value of the maximum run concentration ratio. The maximum run concentration ratio observed in figure 3b can be explained as follows: at low run flow ratios, the run concentration ratio is high, especially for the coarse particles. This is because the coarse particles have a higher particle inertia parameter which causes the particles to move in straight lines and not to follow the fluid streamlines into the branch. This segregation is also augmented by the drastic variation in the vertical concentration profile for the coarse particles. As the run flow ratio increases, more fluid reports to the run and consequently, the run concentration ratio decreases.

Figures 4a and 4b show the effect of the upstream bulk velocity on the concentration ratios for the medium sand at a solids concentration of 12%. The upstream bulk velocity affects the branch and the run solids concentrations as follows: as the bulk velocity is increased, the particle inertia parameter, K, becomes larger and the concentration profile upstream of the T-junction becomes more uniform. The former would cause lower branch concentration ratios and higher run solids concentration ratios. The latter would result in opposite trends. The results shown in figure 4a indicate that the net effect of having a higher upstream bulk velocity is an increase in the branch solids concentration. This implies that the effect of modifying the solids concentration profile as a result of increasing the bulk velocity is more significant than that of increasing the particle inertia parameter. One also notes from figure 4b that the maximum value of C_2/C_0 gradually decreases as the upstream bulk velocity becomes higher.



Figure 4a. Effect of upstream bulk velocity on the branch concentration ratio for upward orientation for $d_{50} = 0.23$ mm and $C_0 = 12\%$.



Figure 4b. Effect of upstream bulk velocity on the run concentration ratio for upward orientation for $d_{50} = 0.23$ mm and $C_0 = 12\%$.





Figure 5a. Effect of upstream solids concentration on the branch concentration ratio for upward orientation at $d_{50} = 0.23$ mm and $U_0 = 2.6$ m/s.

Figure 5b. Effect of upstream solids concentration on the run concentration ratio for upward orientation at $d_{s0} = 0.23$ mm and $U_0 = 2.6$ m/s.

The effect of the upstream solids concentration on the concentration ratios is of interest, since the upstream concentration profile and the drag force exerted by the fluid on the particles are functions of the solids concentration. Figures 5a and 5b show the effect of the upstream solids concentration on the concentration ratios for the medium sand at a bulk velocity of 2.6 m/s. One observes in figure 5a that for a given flow ratio, a higher upstream solids concentration gives a higher branch solids concentration. This trend is reasonable since by increasing the solids concentration, the concentration profile becomes more uniform and the particle drag becomes larger. This results in a higher particle acceleration leading to a higher branch solids concentration. One also observes from figure 5b that the maximum run concentration ratio decreases as the upstream solids concentration increases.

Downward orientation

Figures 6a and 6b illustrate the effect of flow ratio on the concentration ratios C_1/C_0 and C_2/C_0 , respectively, with the particle size as a parameter. Slurries with solids concentration of 12% and a bulk velocity of 2.6 m/s were used with a T-junction with a downward orientation. Unlike the branch concentration ratio for the upward orientation shown in figure 3a and the previous results for branches of vertical approach (Nasr-El-Din & Shook 1986), C_1/C_0 for downward orientation is mainly greater than unity, whereas C_2/C_0 is always less than one. As shown in figure 6a, the branch concentration ratio is zero at a flow ratio of zero. As the flow ratio is increased from zero, the branch concentration ratio increases rapidly to a maximum (greater than unity), then gradually decreases to one as the flow ratio approaches unity. The value of the maximum depends on the particle size. The largest particle size gives the highest C_1/C_0 value. A comparison between figures 6a and 3a shows that the fine sand is the least affected by changing the branch orientation.





Figure 6a. Effect of particle size on the branch concentration ratio for downward orientation at $C_0 = 12\%$ and $U_0 = 2.6$ m/s.

Figure 6b. Effect of particle size on the run concentration ratio for downward orientation at $C_0 = 12\%$ and $U_0 = 2.6$ m/s.

This result is reasonable because the fine particles have low particle inertia parameter and have a more uniform concentration profile ahead of the T-junction. The effect of the branch orientation is greatest for the coarse particles. The difference in the results for the two orientations is mainly attributed to the solids concentration profile upstream of the T-junction and to the effect of the gravity force, as explained below.

Flow visualization for the coarse sand, especially at low bulk velocities, has shown that there are distinct upper and lower layers. The upper layer consists mainly of the carrier fluid, whereas solids are more concentrated in the lower layer. As a result, one expects that the average velocity in the upper layer to be much higher than that in the lower section of the pipe. Previous velocity measurements (Brown & Shook 1982) have shown that a maximum velocity occurs somewhere in the upper section of the pipe. This implies that the particle inertia parameter is high in the upper layer, and low in the lower layer. The high solids concentration in the lower section of the pipe implies that the drag force is much higher in this section leading to higher branch concentrations. For the upward orientation, particles move against the gravity force, whereas for the downward orientation they move in the same direction as the gravity force. However, this is a minor effect, especially at high bulk velocities as discussed earlier.

In the limiting case where all the solids report to the branch, i.e. the run solids concentration equals zero, the branch solids concentration can be calculated as

$$\frac{C_1}{C_0} = \left(\frac{Q_1}{Q_0}\right)^{-1}.$$
[11]

It is interesting to note that unlike the results shown in figure 3b, the branch concentration ratio for the downward orientation (figure 6a) is far from the total solids separation limit. This indicates the importance of the upstream conditions and the branch orientation on the branch concentration ratio. Figure 6b shows that the run concentration ratio is less than unity. This indicates that the effect of a downward T-junction is to lower the solids concentration downstream of the T-junction.

Figures 7a and 7b show the effect of the upstream bulk velocity on the branch and run solids concentration ratios for the medium sand at an upstream solids concentration of 12%. One observes that at a given flow ratio as the upstream bulk velocity is increased, C_1/C_0 decreases whereas C_2/C_0 increases. These trends are opposite to those which occurred for the upward orientation shown in figures 4a and 4b. Of course, when the bulk velocity is higher, the solids concentration profile becomes more uniform and this causes the branch solids concentration to decrease as explained earlier. In addition, the particle inertia parameter increases as the upstream bulk velocity increases. Both factors would cause the branch concentration to be lower, as shown in figure 7a.

Figures 8a and 8b show the effect of the upstream solids concentration on the concentration ratios at a bulk velocity of 2.6 m/s. For a given flow ratio, one observes that the run and the branch concentrations are affected differently by the upstream solids concentration. This result can be explained as follows: as the upstream solids concentration increases, the concentration profile



Figure 7a. Effect of upstream bulk velocity on the branch concentration ratio for downward orientation for $d_{50} = 0.23$ mm and $C_0 = 12\%$.



Figure 7b. Effect of upstream bulk velocity on the run concentration ratio for downward orientation for $d_{50} = 0.23$ mm and $C_0 = 12\%$.





Figure 8a. Effect of upstream solids concentration on the branch concentration ratio for downward orientation for $d_{50} = 0.23$ mm and $U_0 = 2.6$ m/s.



becomes more uniform, i.e. the ratio of the solids concentration of the lower layer relative to the total solids concentration decreases. Consequently, the velocity of the lower layer and hence the particle inertia parameter increases as a result of increasing the upstream solids concentration. Both factors would cause the branch solids concentration to decrease. Although [5] predicts a similar trend to that shown in figure 8a, the effect of solids concentration on the branch concentration ratio is more complicated than that predicted by [5]. According to [5], the only effect of the upstream solids concentration is due to C_{bottom}/C_0 , whereas in reality, the particle inertia also changes because of the change in the concentration profile, especially in the lower half of the pipe. This effect was not included in [5].

Side orientation

Measuring the branch and run solids concentration ratios for the side orientation was of interest, since unlike the upward and downward orientations the effects of gravity and the concentration profile upstream of the T-junction are insignificant. Figures 9a and 9b show the effect of the particle size on the branch and run solids concentration ratios for an upstream solids concentration of 12% and a bulk velocity of 2.6 m/s. As can be seen in figures 9a and 9b the branch concentration ratio is less than unity, whereas the run concentration ratio is higher than unity. The branch and the run solids concentrations for the side orientation are also functions of the particle size. For a given flow ratio, as the particle size increases, the branch concentration ratio decreases, and the run concentration ratio is necesses. This result is reasonable because as the particle size increases the particle inertia parameter increases and this lowers the branch concentration. It appears from the results shown in figures 3a, 6a and 9a that the effect of the particle size on the branch solids concentration is much less than those for the other two orientations. The total



Figure 9a. Effect of particle size on the branch concentration ratio for side orientation at $C_0 = 12\%$ and $U_0 = 2.6$ m/s.

Figure 9b. Effect of particle size on the run concentration ratio for side orientation at $C_0 = 12\%$ and $U_0 = 2.6$ m/s.





Figure 10a. Effect of upstream bulk velocity on the branch concentration ratio for side orientation for $d_{50} = 0.23$ mm and $C_0 = 12\%$.

Figure 10b. Effect of upstream bulk velocity on the run concentration ratio for side orientation for $d_{50} = 0.23$ mm and $C_0 = 12\%$.

solids separation line shown in figure 9b corresponds to zero solids concentration in the branch. Unlike the run concentration ratio for the upward orientation shown in figure 3b, the run concentration ratio for the three sand fractions deviate significantly from the total solids separation limit.

Figures 10a and 10b show the effect of the upstream bulk velocity on the branch and run solids concentrations ratios for the medium sand at an upstream solids concentration of 12%. At a given flow ratio, as the upstream bulk velocity increases, the branch concentration ratio decreases and the run concentration ratio increases. The effect of the bulk velocity on the branch concentration ratio for side orientation shown in figure 10a is due to particle inertia effect. For a higher bulk velocity, the particle inertia parameter increases, leading to lower branch concentration ratios.

Solids transport efficiency

Another parameter of interest is the solids distribution in the branch and the run. Figure 11 shows the branch solids transport efficiency (E_1) , defined by [2], for the coarse particles for the three orientations at a solids concentration of 12% and a bulk velocity of 2.6 m/s. The line $E_1 = E_2$ corresponds to equal solids distribution between the branch and the run. Figure 11 shows that the branch orientation plays an important role in determining the solids distribution. One observes that the side orientation gives the best solids distribution of the three orientations studied. It also appears from figure 11 that the downward orientation gives a relatively better solids distribution than the upward one.

Figure 12 shows the effect of the upstream bulk velocity on the branch transport efficiency for the upward and downward orientations for the medium sand at an upstream solids concentration of 12%. For a given flow ratio, as the upstream velocity is increased the transport efficiency increases for the upward orientation and decreases for the downward orientation. In other words,



Figure 11. Effect of branch orientation on the branch transport efficiency for $d_{s0} = 0.40$ mm at $C_0 = 12\%$ and $U_0 = 2.6$ m/s.



Figure 12. Effect of bulk velocity on the branch transport efficiency for upward and downward orientations at $d_{so} = 0.23$ mm and $C_0 = 12\%$.

Figure 13. Effect of concentration on the branch transport efficiency for upward and downward orientations at $d_{s0} = 0.23$ mm and $U_0 = 2.6$ m/s.



a higher upstream bulk velocity improves solids distribution for both orientations. One also observes that the downward orientation has a better solids distribution than that of the upwards orientation, especially at high upstream bulk velocities.

Figure 13 shows the effect of the upstream solids concentration on the branch transport efficiency for the medium sand at a bulk velocity of 2.6 m/s using branches with upward and downward orientations. For a given flow ratio, as the upstream solids concentration increases the branch transport efficiency (E_1) approaches the diagonal, i.e. E_1 approaches E_2 . A comparison between figures 12 and 13 shows that the effect of the upstream solids concentration on the branch transport efficiency is similar to that for the upstream bulk velocity. It is interesting to note that a T-junction with a downward orientation distributes solids better than that with an upward orientation, especially at high solids concentration or at high upstream bulk velocity.

The effects of the upstream bulk velocity and solids concentration on the branch transport efficiency for the side orientation are different from those of upward or downward orientations. Figure 14 shows the effects of solids concentration on the branch transport efficiency for the side orientation. One observes that the effect of solids concentration on the transport efficiency is almost insignificant within the range of solids concentration examined in this study. The branch transport efficiency was found to decrease slightly as the upstream bulk velocity increases for side orientation. This is due to the effect of the particle inertia parameter explained earlier.

CONCLUSIONS

- 1. The branch and the run concentration ratios are functions of the flow ratio. The concentration ratios depend on the branch orientation: for the upward and the side orientations, the branch concentration ratio is always less than unity. The run concentration ratio exhibits a maximum at a run flow ratio of about 0.1. For the downward orientation, branch concentration ratio is mainly greater than unity, but the run concentration ratio is always less than one.
- 2. For all orientations, the branch and the run concentration ratios are functions of the upstream conditions. For the vertical upward orientation and at a given flow ratio, the branch concentration ratio increases as the upstream velocity or solids concentration increases, but decreases as the particle size increases. The opposite trends occur for the vertical downward orientation. For side orientation, the branch concentration ratio was found to be independent of the upstream solids concentration, but decreases as the upstream bulk velocity or particle size becomes larger.
- 3. Regarding the transport efficiencies for the branch and the run, the side orientation gives nearly the best solids distribution, especially at low upstream bulk velocities. At higher upstream bulk velocities, nearly equal solids distribution was obtained with the downward orientation.





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